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SPECTRAL, NOISE AND CORRELATION PROPERTIES OF INTENSE SQUEEZED LIGHT GENERATED BY A COUPLING IN TWO LASER FIELDS

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Abstract

Two schemes of four-wave mixing oscillators with nondegenerate pumps are proposed for above-threshold generation of squeezed light with nonzero mean-field amplitudes. Noise and correlation properties and optical spectra of squeezed-light beams generated in these schemes are discussed.

1. Introduction

The squeezed light generated to date has been in the main either squeezed vacuum or squeezed light with an extremely small mean field amplitude. Therefore there is much current interest in searching of new schemes to generate squeezed light with a large coherent component of the field.

In this report we would like to present some nonlinear optical schemes for generation of intense squeezed light with nonzero mean amplitude. This type of coherent squeezed-state light is called sometimes bright squeezed light.

One of the important schemes for the generation of squeezed light, realized experimentally, is based the process of nondegenerate four-wave mixing (FWM) in a cavity. In this process an intense pump field with frequency ω_0 (for certainty) interacts via a nonlinear medium with two modes of radiated field with frequencies ω_1, ω_2, such that $2\omega_0 \to \omega_1 + \omega_2$.

In contrast with this standard scheme of FWM, we propose to consider the process of FWM under the pumping by two laser fields of different frequencies ω_1, ω_2. As a result of coupling in nonlinear

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medium, a pair of photons of two pump fields ω_1, ω_2 transform to a pair of photons of spontaneously generated signal field with degenerate frequency ω_0, such that $\omega_1+\omega_2 \rightarrow 2\omega_0$.

We study two different four-wave mixing configurations. The first of them (see Fig.1) consists of a nonlinear medium in a ring cavity, which couples two monochromatic copropagating pump beams of different colors (frequencies ω_1, ω_2) with an intracavity signal mode at the half-sum frequency $\omega_0=(\omega_1+\omega_2)/2$. We consider the case of spontaneous excitation of a single cavity resonant mode. So we have dealing with a nearly collinear wave-vector matching condition $\vec{k}_1+\vec{k}_2 \simeq 2\vec{k}_0$.

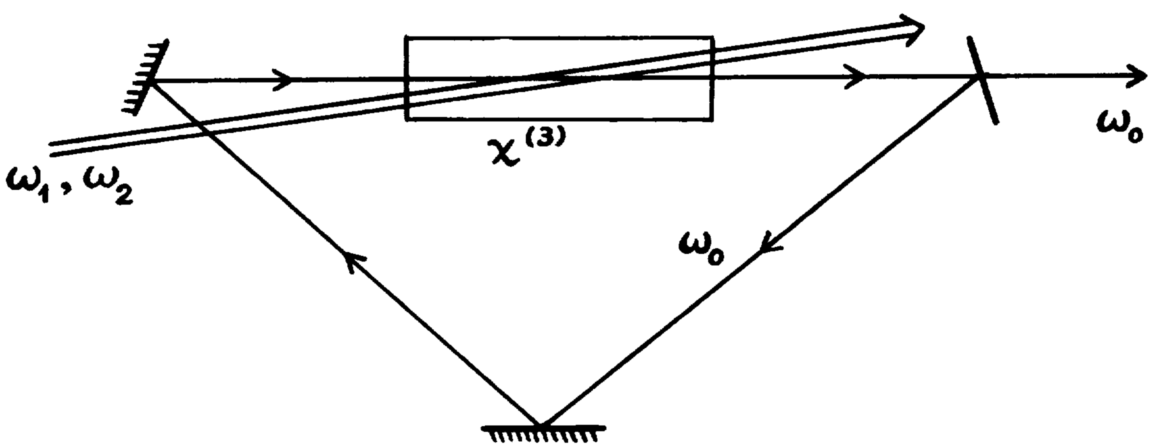

Fig.1. Scheme of the double-color-pumped FWM oscillator with a single cavity-mode excitation.

In the first part of our report (see Section 2) we shall present the results, related to the configuration of Fig.1. At the beginning of this part we would like to describe briefly some of the below threshold results [1,2].

2.1. Squeezing of the central line of the resonance fluorescence in a bichromatic field in a cavity

We consider an ensemble of two-level atoms interacting in an optical cavity with a bichromatic pump field and with a cavity mode of radiation field. This pump field is treated classically and chosen in the following form

$$E(t) = E_0 \ \mathrm{Re}\left[e^{-i(\omega_0+\delta)t-2i\phi} + e^{-i(\omega_0-\delta)t} \right] . \tag{1}$$

It contains two components with equal amplitudes $E_0/2$, relative phase 2ϕ and frequencies $\omega_{1,2}=\omega_0\pm\delta$, symmetrically detuned from the atomic transition frequency ω_0.

We have calculated the cavity-output intensity in the process of resonance fluorescence [1]. At this we do not require the execution of any phase-matching conditions. The result for the inelastic part of the intensity is shown in Fig.2 as a function of the detuning of the cavity resonance frequency ω_c from the atomic transition frequency ω_0. The curve is plotted for particular values of the pump intensity parameter $\xi=2V/\delta$ and parameters Γ/σ, δ/γ (V is the matrix element of the dipole transition, σ is the atomic absorption coefficient, γ is the atomic spontaneous wight, Γ is the cavity decay rate)

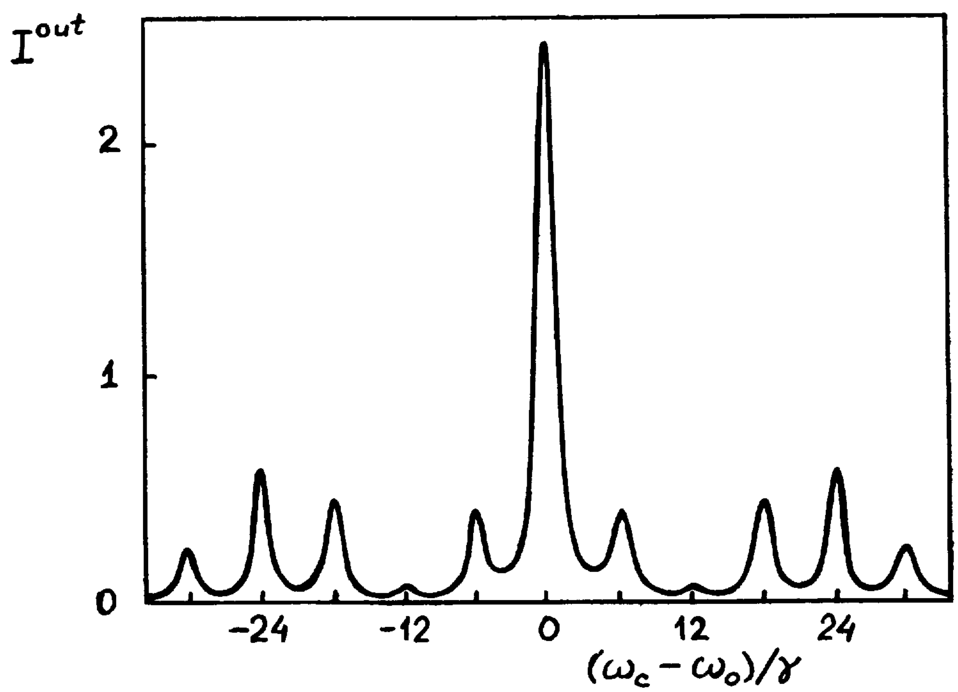

Fig.2. Cavity output intensity versus $(\omega_c-\omega_0)/\gamma$: $\xi=5$, $\delta/\gamma=6$, $\Gamma/\sigma=0,1$.

We see that the intensity consist of a series of peaks with a constant spacing δ. They are symmetrically located about a central peak, coinciding with the atomic transition frequency ω_0. This intensity spectrum was experimentally studied by Y.Zhu et al. [3] for two-level-like Ba atoms driven by two strong, equal-amplitude fields with frequency separation 2δ. Note that the results of our calculations are in agreement with the experimental curves.

Let us turn now to the results, related to the quantum fluctuations in the process of FWM. We consider a generation of the mode with frequency equal to the resonance fluorescence central line $\omega_0=(\omega_1+\omega_2)/2$. The calculation of the quadrature-phase fluctuations show that this ω_0-mode is excited in a squeezed state. The optimal value of the squeezing spectrum $S(\omega)$ is realized at zero frequency and the maximal squeezing may reach 35%. The dependence of the quantity $S_{min}(\omega=0)$ on the pump intensity parameter $\xi=2V/\delta$ is plotted in Fig.3 [2].

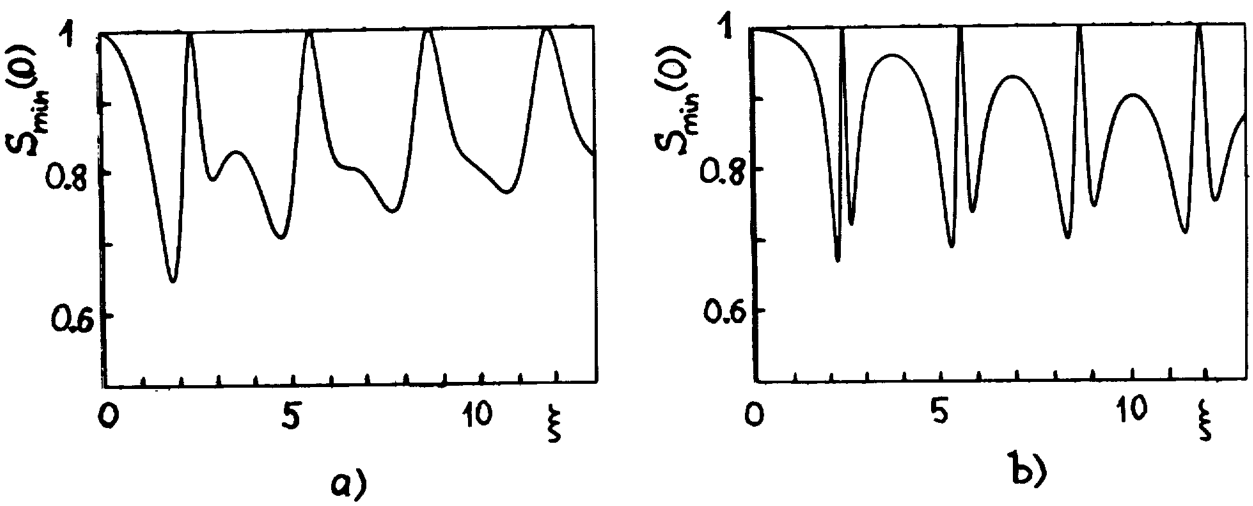

Fig.3. Peak squeezing $S_{min}(0)$ versus ξ for $\Gamma/\sigma=0.1$ - (a), $\Gamma/\sigma=0.01$ - (b). The squeezing is absent for values of ξ, for which the probabilities of one and twin ω_0-photon radiation processes cancell each other.

In order to interpret this result, note that the excitation of the ω_0-mode in a cavity is caused by a nonlinear spontaneous process of two-photon radiation by an atom in a bichromatic field. In a low order of interaction it may be represented by the following graphs (see Fig.4). It is essential that there is a strong pair correlation between the photons of frequency ω_0. This correlation has a superbunching behavior and manifests itself in the reduction of quadrature fluctuations below the shot-noise level.

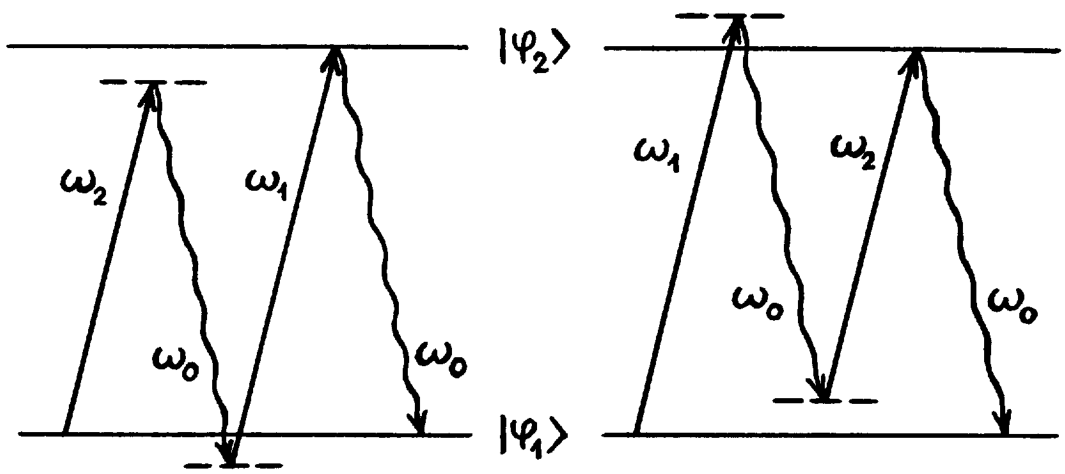

Fig.4. Illustration of the process of two-photon absorption at frequencies $\omega_{1,2}=\omega_0\pm\delta$ of the pump fields with the emission of two photons at frequency $\omega_0=(\omega_1+\omega_2)/2$.

2.2. Above threshold results in a parametric model of FWM in the presence of phase modulation

Now we consider a simple parametric model of four-wave mixing under bichromatic pumping in order to obtain the squeezing results above the generation threshold. In our analysis we include the effects of self-phase modulation and cross-phase modulation.

We describe the nonlinear medium phenomenologically by the third order susceptibility $\chi^{(3)}$. So we start from the following Hamiltonian

$$H = \hbar\omega_c a^+ a + \frac{\hbar\chi}{2}\, a^2 E_1^* E_2^* + a^{+2} E_1 E_2 + \frac{\hbar\chi}{4}\, a^{+2} a^2 +$$

$$+ \hbar\chi\left[\,|E_1|^2+|E_2|^2\right] a^+ a + (a^+\Gamma + a\Gamma)\ , \qquad (2)$$

where a^+, a are creation and annihilation operators of the intracavity mode, ω_c is the cavity resonant frequency. The second term in Eq.(2) describes the four-wave interaction with coupling constant χ, proportional to $\chi^{(3)}$. The third and forth term describe the self-phase modulation and cross-phase modulation. The fifth term accounts for the coupling of the cavity mode with reservoir, which will give rise to the cavity damping constant γ. The quantities E_1, E_2 are the complex amplitudes of the pump fields at frequencies ω_1, ω_2. At nearly collinear phase matching condition these pump fields generate an intracavity signal mode with frequency $\omega_0=(\omega_1+\omega_2)/2$. In this configuration the pump fields travel through the medium only once. So it is possible to neglect the pump depletion and treat E_1, E_2 as a fixed constants.

With use of standard methods, we obtain the following differential equations for the stochastic amplitude of the signal mode ω_0:

$$\frac{d\alpha}{dt} = -\gamma\alpha + i\left[\Delta-\chi\left\{|E_1|^2+|E_2|^2\right\}\right]\alpha - \frac{i\chi}{2}\alpha^+\alpha^2 - i\chi E_1E_2\alpha^+ + R_0(t), \quad (3)$$

where $\Delta=\omega_0-\omega_c$ is the cavity detuning parameter. The noise term R_0 has the following correlator

$$\langle R_0(t)R_0(t')\rangle = -\chi\left[E_1E_2 + \frac{1}{2}\alpha^2\right]\delta(t-t') \quad (4)$$

Note that without the phase-modulation terms equations of motion would be the same as for the degenerate parametric oscillator and degenerate four-wave mixing below threshold. The novel features and results in our system are connected with the incorporation of the self-phase modulation term. This term results in the above threshold generation of the signal field. Let us consider the stable steady-state solution for the output intensity. It is equal to

$$I^{out} = 2\gamma\langle\alpha^+\alpha\rangle = 2\gamma\left[\frac{\Delta}{\gamma} - \frac{\chi}{\gamma}\left(|E_1|^2+|E_2|^2\right) + \sqrt{\left(\frac{\chi}{\gamma}\right)^2|E_1|^2|E_2|^2-1}\right]. \quad (5)$$

In Fig.5 we plot the normalized output intensity as a function of the parameter $\varepsilon^2=E^2\chi/\gamma$ for the case of equal amplitudes of the pump fields $|E_1|=|E_2|=E$.

The zero intensity solution is stable below the generation threshold at $\varepsilon^2<\varepsilon_A^2$ and well above-threshold at $\varepsilon^2>\varepsilon_B^2$, where

$$\varepsilon_A^2 = \frac{1}{3}\left[\frac{2\Delta}{\gamma} - \sqrt{\left(\frac{\Delta}{\gamma}\right)^2-3}\right], \quad \varepsilon_B^2 = \frac{1}{3}\left[\frac{2\Delta}{\gamma} + \sqrt{\left(\frac{\Delta}{\gamma}\right)^2-3}\right]. \quad (6)$$

The nonzero-intensity solution given by Eq.(5) is stable in the region $1<\varepsilon^2<\varepsilon_B^2$ and have a meaning for Δ less than $\gamma\sqrt{3}$. We see that a bistable behaviour of the output is realized in the region $1<\varepsilon^2<\varepsilon_A^2$.

Now let us turn out to the problem of squeezing. The spectrum of the output field above threshold is obtained in a standard linearized treatment of quantum fluctuations. Curves for squeezing spectra $S(\omega)$ versus ω/γ and the spectral value $S(\omega_{opt})$ at the points of optimal frequency $\omega=\omega_{opt}$ versus the pump field intensity parameter $\varepsilon^2=E^2\chi/\gamma$ ($E=|E_1|=|E_2|$) are plotted in Fig.6 (a),(b).

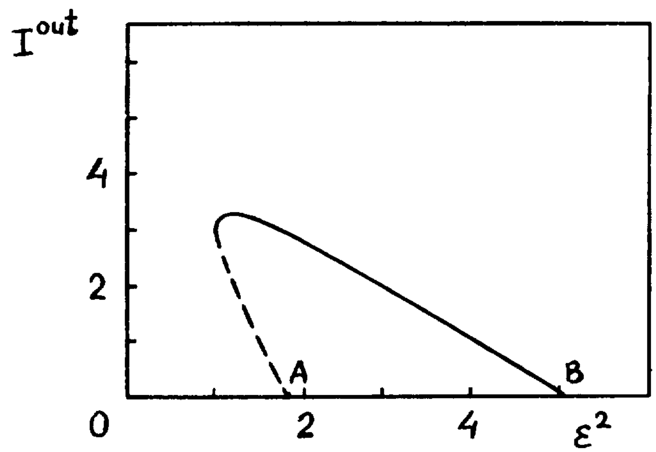

Fig.5. Output intensity of the FWM oscillator versus the intensity of the pump fields: $\Delta/\gamma=5$.

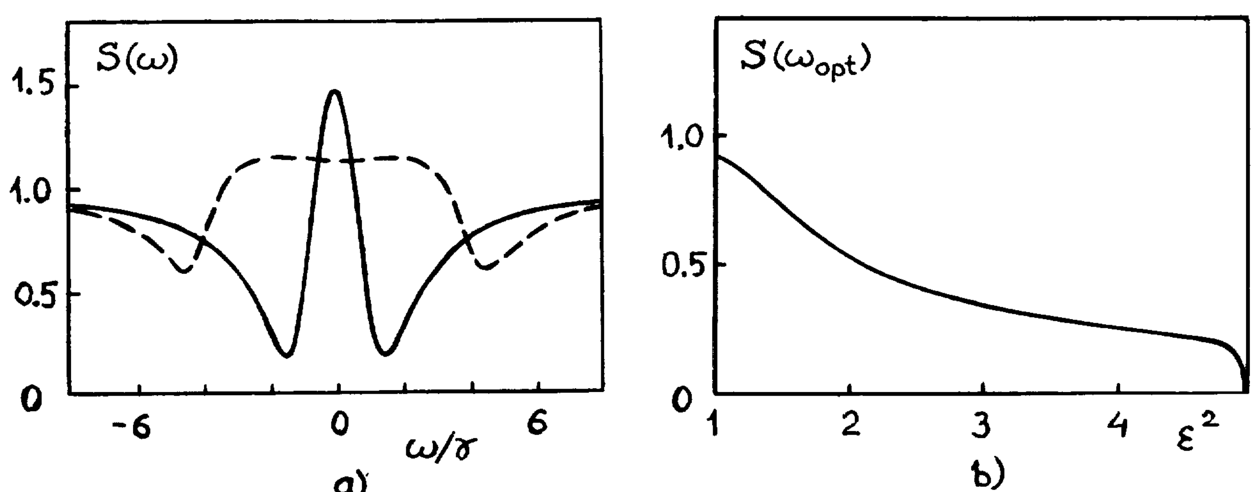

Fig.6. (a) - Squeezing spectrum versus ω/γ for $\Delta/\gamma=5$, $\varepsilon^2=1.8$ (dotted line) and $\varepsilon^2=4.8$ (solid line); (b) - dependence of the quantity $S(\omega_{opt})$ on ε^2 for $\Delta/\gamma=5$.

It should be noted that the experimental measurement of the noise on the qudrature component in a similar FWM configuration has been carried out by D.Grandclement et al [4]. They did not find the squeezed noise reduction. This is not so surprising because, as follows from our analysis, squeezing is realized for properly chosen values of parameters Δ/γ and ε^2.

3. Intracavity parametric FWM with nondegenerate pumps. Above-threshold results on bright squeezing of the three intracavity modes

This part of our report is devoted to the results on bright squeezing in the configuration of FWM, where the two laser driving fields propagate in the direction of the cavity axis (see Fig.7). These driving fields feed two intracavity pump modes at the frequencies ω_1, ω_2. The pump modes generate a signal mode at their half-sum frequency $\omega_0=(\omega_1+\omega_2)/2$ and the wave-vector matching condition are executed exactly ($\vec{k}_1+\vec{k}_2=2\vec{k}_0$). So the configuration of FWM with tree intracavity resonant modes of frequencies ω_1, ω_2 and ω_0 is realized.

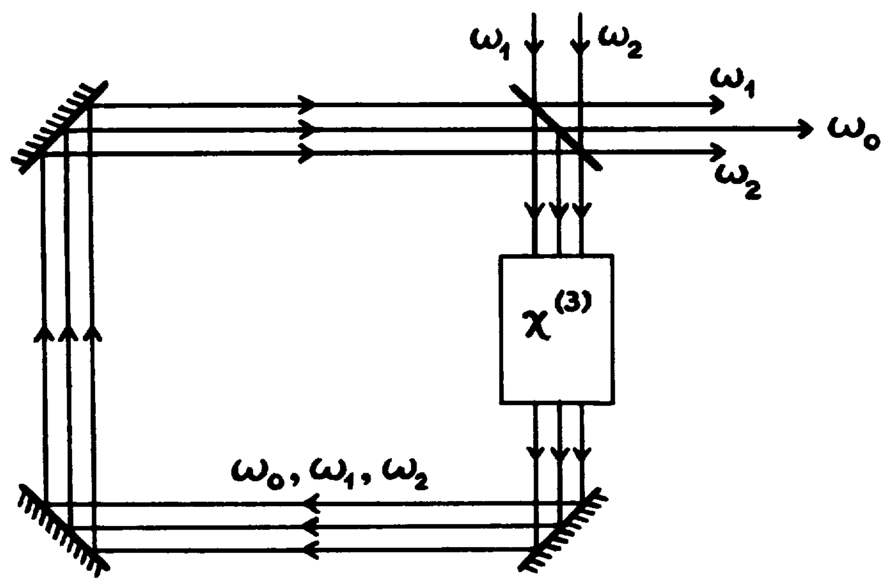

Fig.7. Scheme of the double-color-pumped FWM oscillator with three intracavity modes ω_1, ω_2, ω_0 ($\omega_1+\omega_2=2\omega_0$).

In this configuration the effects of mutual influence of the pump and signal modes are essential. So we take into account the pump depletion. The consideration is simplified however in that we ignore the phase modulation terms.

Note that the advantages of this scheme of FWM, as compared to the standard nondegenerate FWM with a single pump mode, are caused by the following. As shown below the effect of phase diffusion is absent here. As a result, the output field for each of the three

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modes have nonzero mean amplitudes with a definite phases.

Thus we start from the following Hamiltonian

$$H = \sum_{j=0}^{2} \hbar\omega_j a_j^+ a_j + i\hbar \frac{\chi}{2} \left[a_1 a_2 a_0^{+2} - a_1^+ a_2^+ a_0^2\right] +$$

$$+ i\hbar \sum_{k=1}^{2} \left[E_k e^{-i\omega_k t} a_k^+ - E_k^* e^{i\omega_k t} a_k\right] + \sum_{j=0}^{2} \left[a_j^+ \Gamma_j + a_j \Gamma_j^+\right] \tag{7}$$

As compared to the previous model, now we take into account the quantization of the pump modes and incorporate: (i) the coupling of the pump modes with two external coherent driving fields of amplitudes E_1, E_2 and (ii) the decay of the three cavity modes due to the coupling with reservoirs.

With use of standard methods, a Fokker-Planck equation in positive P-representation for the system is found, from which stochastic differential equations for the complex field amplitudes are obtained

$$\begin{aligned}
\dot{\alpha}_0(t) &= -\gamma_0\alpha_0 + \chi\alpha_1\alpha_2\alpha_0^+ + R_0(t) \\
\dot{\alpha}_1(t) &= -\gamma\alpha_1 - \frac{1}{2}\chi\alpha_0^2\alpha_2^+ + E\exp(i\phi_1) + R_1(t) \\
\dot{\alpha}_2(t) &= -\gamma\alpha_2 - \frac{1}{2}\chi\alpha_0^2\alpha_1^+ + E\exp(i\phi_2) + R_2(t)
\end{aligned} \tag{8}$$

Here γ_0, $\gamma\equiv\gamma_1=\gamma_2$ are the damping constants for the modes ω_0 and ω_1, ω_2 respectively, E is the amplitude of the driving fields $E_{1,2}=E\exp(i\phi_{1,2})$, and ϕ_1, ϕ_2 are arbitrary phases of the driving fields. R_j are Gaussian noise terms with the following nonzero correlations

$$\langle R_0(t)R_0(t')\rangle = \chi\alpha_1\alpha_2\delta(t-t'), \quad \langle R_1(t)R_2(t')\rangle = -\frac{\chi}{2}\alpha_0^2\delta(t-t'). \tag{9}$$

In order to analyze the quantum fluctuations of the modes we apply a linearized treatment of fluctuations about the stable steady-state solutions. It is worth noting that, as opposed to the standard scheme of FWM, for the present system there exist three types of stable steady-state solutions. They correspond to the three possible regimes of oscillation: one below the generation threshold $E<E_t$ and two different above-threshold regimes at $E_t<E<2E_t$ and $E<2E_t$. The threshold value of the amplitude E is $E_t= =\gamma(\gamma_0/\chi)^{1/2}$.

The results for the cavity-output intensities N_j^{out} (in photon number units per unit time) for the modes ω_j in the above-threshold regime are following. In the region $1<\varepsilon<2$ ($\varepsilon\equiv E/E_t$) we have [5,6]:

$$N_0^{out} = \frac{4\gamma\gamma_0}{\chi}(\varepsilon-1) \quad , \quad N_1^{out}=N_2^{out}= \frac{2\gamma\gamma_0}{\chi}\left[1-\frac{\varepsilon}{2}\right]^2 \quad ; \qquad (10.a)$$

and in the region $\varepsilon>2$:

$$N_0^{out} = \frac{4\gamma\gamma_0}{\chi} \quad , \quad N_1^{out}=N_2^{out} = \frac{2\gamma\gamma_0}{\chi}\left(\frac{\varepsilon^2}{2}-1\right) \qquad (10.b)$$

The steady-state phases ψ_j of all the three modes are defined above threshold and equal to

$$\psi_1=\phi_1 \quad , \quad \psi_2=\phi_2 \quad , \quad \psi_0=(\phi_1+\phi_2)/2 \qquad (11)$$

So, they are determined by the phases of the driving fields ϕ_1, ϕ_2.

Squeezing spectra above threshold

We calculate the quadrature fluctuation variance for all the three intracavity modes and corresponding squeezing spectra for the cavity-output fields. We would like to point out at once that an effective squeezing occurs for each of the three modes above threshold [5,6]. This is an extremely interesting feature of the double-color-pumped FWM oscillator.

The maximal squeezing is realized for the phase of the local oscillator equal to $\vartheta_j=\psi_j+\frac{\pi}{2}$ and is determined by the fluctuations of phase variables

$$S_j(\omega) = 1 + 8\gamma_j n_j \langle\delta\psi_j(-\omega)\delta\psi_j(\omega)\rangle \quad , \qquad (12)$$

where n_j is the intracavity steady-state photon number of the ω_j-mode.

Examples of the curves of the squeezing spectrum for the signal mode are plotted in Fig.8 for different values of parameters ε and $r=\gamma_0/\gamma$.

Our analysis show that a noise reduction below the shot noise level may reach approximately 100% in the whole above-threshold region $\varepsilon>1$ and for $r\geq10$. The higher γ_0/γ the better the squeezing. However, the intensity of this field is limited by the value $N_0=4\gamma_0\gamma/\chi$. Generation of more intense light in the squeezed state occurs at the pump field frequencies. The corresponding output intensities grow with increase of the incident fields. However the maximal squeezing may reach approximately 50% for certain values of the ratio γ_0/γ.

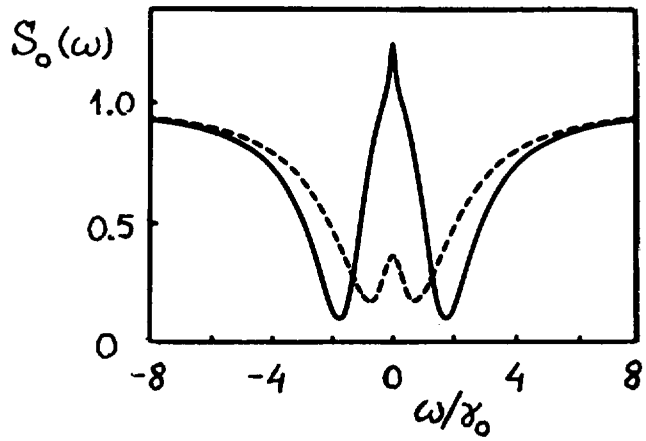

Fig.8. Squeezing spectrum $S_0(\omega)$ versus ω/γ_0: ε=1.1, r=2 - (solid line); ε=4, r=10 - (dotted line).

We restrict ourselves by representation of the squeezing spectrum $S_{1,2}(\omega_{opt})$ at the points of minima $\omega=\omega_{opt}$. The dependence of this quantity on the parameter r is plotted in Fig.9.

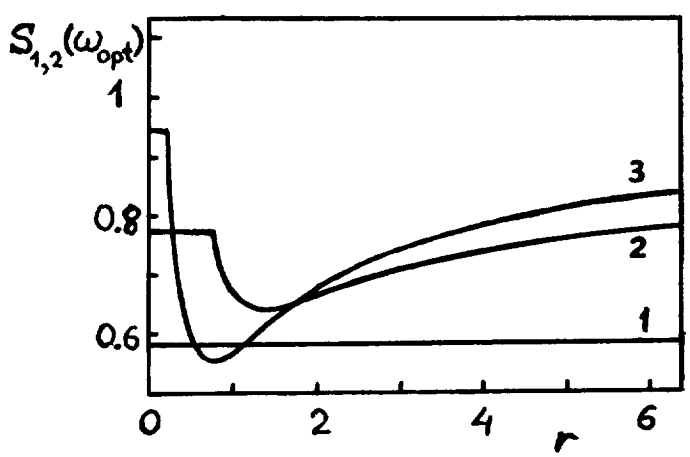

Fig.9. Dependence of the quantity $S_{1,2}(\omega_{opt})$ on r: (1) ε=2.2, (2) ε=2; (3) ε=6.

Thus we find that the double-color-pumped FWM oscillators is extremely promising for the above-threshold generation of one-mode bright squeezed light.

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Now we shall present the results on the sub shot-noise correlations in the above-threshold regime. We consider the correlations between both the intensities and the phases of the interacting modes.

In the well-known processes of nondegenerate FWM and parametric down-conversion, the photons of two generated modes are created in pairs and a positive correlation $\langle \delta n_1 \delta n_2 \rangle > 0$ between the photon number fluctuations of these modes occurs. It results in the reduction of quantum fluctuations below the shot-noise level in the intensity difference of the modes. This phenomena has been observed for the first time in intracavity nondegenerate parametric oscillator by A.Heidmann et al [7].

In our nonlinear system we have found another manifestation of such an effect. It consists of the reduction of quantum noise in the intensity sum of the pump modes [8]. The explanation of this phenomena is following. The photons of two pump modes are annihilated in pairs and the pump modes acquire correlated statistical properties, which are characteristic for two-photon absorption. As a result, the correlation between initially uncorrelated coherent pump fields becomes negative $\langle \delta n_1 \delta n_2 \rangle < 0$. And this fact results in the sub shot noise fluctuations in the intensity sum of the cavity output beams. We shall not dwell on the particular quantitative results. Note only that the maximal noise reduction may reach approximately 100% in the limit $\varepsilon \to 2$, when the pump depletion is maximal.

A more interesting feature of our FWM configuration is connected with the correlations between the phases of the pump modes. They are studied in terms of the quadrature phase operators, as applied to the twin homodying experimental measurements.

In general the variance of the sum or difference of the quadrature component operator

$$V_{12}^{(\pm)}(\vartheta_1,\vartheta_2) = \left\langle \left[\Delta \left[X_1^{\vartheta_1} \pm X_1^{\vartheta_1} \right] \right]^2 \right\rangle , \quad (X^{\vartheta} = a e^{-i\vartheta} + a^{+} e^{i\vartheta}) \qquad (13)$$

contains the contributions from both the intensity and phase variable fluctuations of the modes.

In our system we can select properly the phases of the local oscillators and to get the variance as expressed in terms of the phase fluctuations only. The result can be written as follows

$$V_{12}^{(\pm)} = 2+4n \langle\delta(\psi_1 \pm \psi_2)^2\rangle \tag{14}$$

Note, that such a possibility do not exist in parametric processes with phase diffusion effect. The nonclassical correlations between the phase fluctuations are manifested in the variance $V_{12}^{(\pm)}$.

In our system the variance of the phase difference fluctuations is negative (in positive P-representation)

$$\langle\delta(\psi_1 - \psi_2)^2\rangle = -\frac{\varkappa}{2\gamma}\frac{\varepsilon-1}{\varepsilon}, \quad (1<\varepsilon<2) \tag{15}$$

and so we obtain a reduction of quantum fluctuations below the shot-noise level in the difference of the quadrature phases: $V_{12}<2$.

A simple analytical result is obtained also for the measured cavity-output fields. The corresponding spectrum of the quadrature phase difference fluctuations in the region $1<\varepsilon<2$ is following

$$S_{12}^{(-)}(\omega)/S_{shot} = 1 + 4\gamma n\langle\delta\psi_1(-\omega)\delta\psi_1(\omega)\rangle + 4\gamma n\langle\delta\psi_2(-\omega)\delta\psi_2(\omega)\rangle -$$

$$- 8\gamma n \operatorname{Re}\langle\delta\psi_1(-\omega)\delta\psi_2(\omega)\rangle = 1 - \frac{4(\varepsilon-1)}{\varepsilon^2+(\omega/\gamma)^2} \tag{16}$$

We see that the noise reduction up to 100% is possible in the limit $\varepsilon\to2$ at zero frequency. In the region $\varepsilon>2$ the shape of the spectrum is complicated and the noise level is increased.

Finally we present some results concerning the optical spectra of the cavity-output squeezed light. We consider the intensity spectra of each of the three nonclassical light beams around the frequencies ω_j $(j=0,1,2)$. These spectra contain a delta-function peak corresponding to coherent part of radiation and a broadened noncoherent part. The broadened parts of spectra are caused by the quantum fluctuations of the field. In the lowest order in quantum fluctuations they contain two contributions, arising from the temporal correlations of the phase and intensity fluctuations $\langle\delta n_j(t+\tau)\delta n_j(t)\rangle$, $\langle\delta\psi_j(t+\tau)\delta\psi_j(t)\rangle$. Depending on the contribution strength, a two- or four-peaked structure of noncoherent part of spectra arise in the case of oscillating character of these correlations.

An example of four-peaked spectrum for the pump field is represented in Fig.10. Here the one pair of the peaks is caused by the phase fluctuations and the other one – by the intensity fluctuations. The fact, that is interesting here, is the separation in frequency of these two contributions. So it seems to be possible to infer the information about the phase fluctuations from the usual optical spectrum.

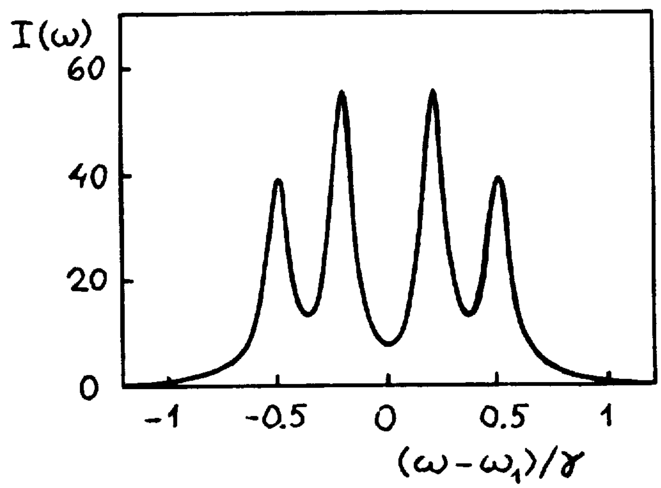

Fig.10. The noncoherent part of the intensity spectrum of the pump mode ω_1: ε=2.2, r=0.05.

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